

TRANSITION RADIATION FROM ROUGH SURFACES

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ABSTRACT. We propose a theory of transition radiation emitted by charged particles entering a rough interface between two media. We show that the radiation process is influenced by surface inhomogeneities distributed both longitudinally and transversally with respect to the direction of particle motion. Moreover, we investigate in detail the transition radiation emitted at normal incidence of particles on a surface with Gaussian distribution of inhomogeneity deviations in relation to the smooth surface.

In the theory of transition radiation, it is assumed that the interface is an ideal plane. Actually, the real surface always presents roughnesses which affect the intensity and polarization of this radiation. The inhomogeneities on a plane surface can vary greatly, and can take the form of individual inhomogeneities insulated from one another, periodically and statistically disposed inhomogeneities. Already in the earliest experiments [1], it was emphasized that the perfection of the finish of the interface greatly influences the polarization of the transition radiation. Detailed experimental reports devoted to this problem have recently been published [2].

As far as investigations of transition radiation enable us to study the optical properties of solids, all the phenomena due to interface roughnesses are of great importance and have to be taken into consideration in the theory of transition radiation. On the other hand, for the study of non-ideal interfaces, there is an opportunity to develop a relatively simple method to control the state of the target just during electron irradiation.

The experimental investigation of roughnesses poses a problem due to their small size and the failure to observe them with an optical microscope. Methods based on the recording of scattered light usually yield parameter estimations averaged over a great number of inhomogeneities. Such an averaging results in a loss of information on the statistics of the assembly of inhomogeneities which vary in size and form and are distributed chaotically.

In optics, resolution is limited by diffraction. Electrons are associated with a wavelength λ given by $\lambda = h/mv$; $m = m_0/\sqrt{1 - \beta^2}$ (m_0 is the electron rest mass, β the ratio of the velocity of the particles and that of light in vacuum, and $h = 6.62 \times 10^{-27}$ erg sec). Owing to the extreme smallness of h , the diffraction of charged particles with energies which are still easily obtainable in practice is weak and is defined by wavelengths which are much smaller than the wavelengths of the optical range.

From the theoretical point of view, the analysis of the influence of interface inhomogeneities on transition radiation is highly complicated. This is caused by a great number of factors which have to be taken into account in order to develop a theory meeting the needs of up-to-date experiment. One of the essential factors are the inhomogeneities caused by physical peculiarities of surface formation which poses the problem of the mathematical description of interface roughnesses. To solve the various problems in media with interfaces it is difficult to satisfy boundary conditions. For every specific case the boundary problem is solved anew. This procedure is awkward and is not applicable practically to the solution of the problem inasmuch as a finite explicit expression for the radiation intensities is not always obtainable. Therefore, there is a need to develop simple approaches to the solution of this type of problems.

The physical picture of radiation on a rough interface is determined by the phenomena arising both in longitudinal and transversal directions of particle motion. There are characteristic quantities, namely, the coherence length and transversal dimensions of the field to be compared with the heights and the transversal size of the roughnesses, the particle "feeling" the surface roughnesses as if the coherent length and transversal dimension of the field were of the same order of magnitude as the longitudinal and transversal size of the roughnesses, respectively.

Consider normal particle incidence onto the target as an example. In the case of an ideally flat interface the momentum transferred to the medium in the course of the radiation process is always perpendicular to the interface. This follows from the homogeneity of the medium in directions parallel to the interface. If the interface is the xy plane, the momentum q_{\parallel} (in reciprocal centimeters) of a particle moving along the z axis with velocity \vec{v} is transferred to the surface only along the motion. In the presence of inhomogeneities, the situation changes and the radiation can transfer both longitudinal and transverse momentum to the interface. As for longitudinal momentum transfer, if it exceeds the "longitudinal momentum" of the surface inhomogeneities, the influence of the inhomogeneities in the longitudinal direction on the transition radiation can be neglected. This situation is well known and has been discussed repeatedly for various processes at higher energies. We shall be interested in effects produced in directions perpendicular to the motion. If a flat interface has a characteristic inhomogeneity of length l in the direction transverse to the motion and the corresponding

momentum uncertainty, the medium can receive momentum of the order of $1/l$ in the transverse direction. This in turn modifies the Ginzburg-Frank equations. How substantial this change will be depends on the contribution made to the transition radiation by momentum transfers of the order $1/l$ in the direction transverse to the motion.

The transverse distances that are effective in the radiation processes are determined by the following expression (see, e.g., Ref. [3])

$$\rho = \frac{\lambda \beta \sqrt{\epsilon}}{\sqrt{1 - \beta^2 \epsilon}} \quad (1)$$

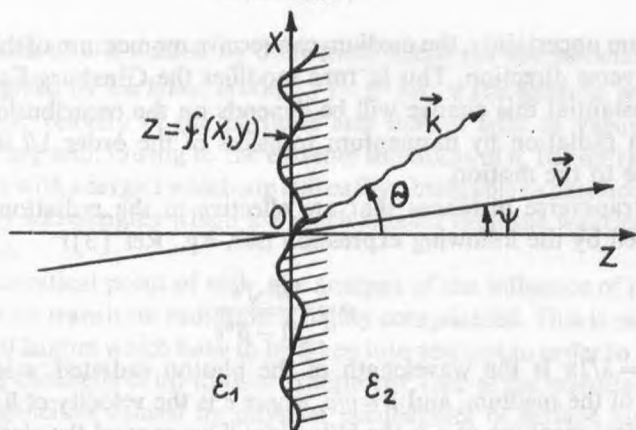
where $\lambda = \lambda/2\pi$ is the wavelength of the photon radiated, ϵ is the dielectric constant of the medium, and $\beta = v/c$, where c is the velocity of light in vacuum. The physical meaning of ρ is the following: if we expand the electric field of the uniformly moving particle in a Fourier integral with respect to time, it turns out that the spectral density of the particle field contains frequencies ω only for collision parameters (the distance from the point at which the particle field is sought to the trajectory in the direction perpendicular to the particle motion) that are smaller than ρ . At larger collision parameters, the spectrum of the particle field contains practically no photons with frequencies exceeding ω .

Since the momentum effectively transferred in a transverse direction in the medium is of the order of $1/\rho$, we can expect the corrections necessitated by transverse effects to be determined by the parameter l/ρ , and these corrections should vanish as $l \rightarrow 0$ and $l \rightarrow \infty$, that is, in the absence of inhomogeneities.

The expression for the energy of the transition radiation from a rough interface will contain parameters that describe the surface. This enables us to study the properties of the surface by using a beam of charged particles.

Using the theory of light scattering on inhomogeneities we propose two mutually complementary methods for the theoretical investigation of transition radiation on a rough interface $z = f(x, y)$. The first method (perturbation theory method) is applicable to the interfaces of two media with only slightly differing refractive indices. The interface, however, can be arbitrary (see, e.g., Ref. [4]). The second method which generalizes the well-known Kirchhoff approximation of the theory of light scattering on a rough surface is applicable to a limited set of interfaces of two media with arbitrary refractive indices (see, e.g., Ref. [5]).

To investigate some general regularities of the transition radiation from arbitrary rough interfaces the perturbation theory method may be conveniently used since it does not specify the surface and gives the opportunity to derive well visualized results. Moreover, to calculate the radiation approximately, an interpolation formula based on the replacement in the final expressions of the factor corresponding to the radiation intensity from the plane interface (calculated with the perturbation theory method) by an exact formula for the transition radiation intensity from the plane interface can be used.



We derive the initial equation for the calculation of the radiation produced when a charged particle crosses the interface $z = f(x, y)$ of two media (see Figure). The function $f(x, y)$ describes small deviations of the interface, due to roughnesses, from the plane $z = 0$; this plane would be the interface in the case of an ideal surface. The particle velocity \vec{v} is directed in the xz plane at an angle ψ to the z axis from the first medium with dielectric constant ϵ_1 to the second medium with dielectric constant ϵ_2 .

To clarify the physical aspect of the problem and to obtain general results without describing specifically the properties of the surface, we use perturbation theory. We construct this theory in analogy with light scattering theory (see, e.g., Ref. [6]), replacing in the latter the scattered wave by the moving particle field which we expand in accordance with the universal procedure in a Fourier integral with respect to time. The radiation problem then reduces to that of scattering of an assembly of monochromatic waves that make up the field of the moving particle. For the scattering effect to be small, it has to be assumed that the dielectric constants of the two media differ insignificantly. A more rigorous test of the validity of the calculations will be given below.

Thus, the perturbation theory calculation developed below for the transition radiation, while not specifying the surface, is applicable to a rather limited group of interfaces between two media, whose refractive indices differ but little. Such are, for example, the interfaces between solid particles and the corresponding immersion liquids, etc. Although the transition radiation yield is proportional to the square of the difference between the refractive indices of the two media, and is consequently strongly suppressed in media with close refractive indices, the calculation method employed makes it possible to cope with the physical picture of the phenomenon and to obtain general formulas that are valid for all interfaces. It is obvious that many qualitative conclusions become valid also for interfaces between two media with greatly differing optical properties.

The radiation energy at large distances R_0 in the frequency interval $d\omega$ and in the solid angle interval $d\Omega$ for an interface of arbitrary shape is determined by the usual expressions of classical electrodynamics with account taken of the dielectric constant of the medium:

$$dI(\omega, \vec{k}) = c \sqrt{\varepsilon_0} |\vec{E}_\omega|^2 R_0^2 d\Omega d\omega \quad (2)$$

where $\varepsilon_0 = (\varepsilon_1 + \varepsilon_2)/2$ is the arithmetic mean of the dielectric constants of the two media. By \vec{E}_ω we denote the intensity of the radiation field of frequency ω at large distances from the interface; this intensity is determined from Maxwell's macroscopic equations (see, e.g., Ref. [3]):

$$\vec{E}_\omega = -\frac{e^{ikR}}{4\pi R} \left[\vec{k} \times \left[\vec{k} \times \int_{-\infty}^{\infty} \vec{E}'_\omega(\vec{r}) e^{-i\vec{k}\vec{r}} \varepsilon'(\vec{r}) d\vec{r} \right] \right], \quad (3)$$

where $\vec{E}'_\omega(\vec{r})$ is the Fourier component of the field of the uniformly moving particle at the point $\vec{r}(x, y, z)$ in a medium with average dielectric constant ε_0 (see, e.g., Ref. [3]):

$$\vec{E}'_\omega(\vec{r}) = \frac{ie}{2\pi^2} \int_{-\infty}^{\infty} \frac{\omega \vec{v}}{k'^2 - \frac{\omega^2}{c^2 \varepsilon_0}} \delta(\omega - \vec{k}'\vec{v}) e^{i\vec{k}'\vec{r}} d\vec{k}', \quad (4)$$

$\varepsilon'(\vec{r})$ is the deviation of the dielectric constant from ε_0 , i.e., in our case

$$\begin{aligned} \varepsilon'(\vec{r}) &= \varepsilon_1 - \varepsilon_0, & -\infty < z < f(x, y), \\ \varepsilon'(\vec{r}) &= \varepsilon_2 - \varepsilon_0, & f(x, y) < z < \infty, \end{aligned} \quad (5)$$

where e is the charge of the electron; the wave vector of the photon emitted is denoted by $\vec{k} = (\omega \sqrt{\varepsilon_0}/c) \vec{n}$ (\vec{n} is a unit vector in the direction of \vec{k}) and the wave vector of the incident pseudophoton is denoted by $\vec{k}'(k'_x, k'_y, k'_z)$.

Integrating (3) with respect to z and using (5), we obtain

$$\begin{aligned} \vec{E}_\omega &= \frac{e(\varepsilon_2 - \varepsilon_1)}{8\pi^3 v_z \varepsilon_0} \frac{e^{ikR_0}}{R_0} \int_{-\infty}^{\infty} \left[\vec{k} \times \left[\vec{k} \times \left(\frac{\omega \vec{v}}{c^2 - \frac{\omega^2}{\varepsilon_0}} \right) \right] \right] \exp[iq_{\parallel} f(x, y)] \\ &\quad \times \exp[i(k'_x - k_x)x + i(k'_y - k_y)y] dk'_x dk'_y dx dy \quad (6) \\ q_{\parallel} &= k'_z - k_z = \frac{\omega - k'_x v_x}{v_z} - k_z. \end{aligned}$$

Substituting (6) into (2) we obtain the following expression for the spectral energy density of the transition radiation:

$$I(\omega, k) = \frac{dI(\omega, k)}{d\Omega d\omega} \quad (7)$$

In the case of a plane interface $z=f(x, y)=0$ we obtain from (6) the conservation law: $k'_x=k_x$ and $k'_y=k_y$. This means that when a photon is emitted in the ϑ direction (ϑ is measured from the z axis, with $0 \leq \vartheta \leq \pi$), momentum is transferred to the interface only in the longitudinal direction, while the transverse momentum carried away by the radiated photon is compensated by the momentum of the pseudophoton. In this case, it is possible to carry out the integration and obtain the transition radiation equations at the plane interface:

$$I_{\text{PL}} = \frac{e^2 |\varepsilon_2 - \varepsilon_1|^2}{4\pi^2 c \varepsilon_0^{3/2}} \beta^2 \sin^2 \vartheta \left| \frac{1 - \beta^2 \varepsilon_0 - \beta \sqrt{\varepsilon_0} \cos \vartheta}{(1 - \beta^2 \varepsilon_0 \cos^2 \vartheta)(1 - \beta \sqrt{\varepsilon_0} \cos \vartheta)} \right|^2. \quad (8)$$

From a comparison of eq. (8) with the exact transition radiation equations it follows that in addition to the condition

$$\left| \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} \right| \ll 1 \quad (9)$$

it is necessary to satisfy one more condition:

$$\left| \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} \right| \ll \frac{\lambda}{l_{\text{coh}}} \cos \vartheta. \quad (10)$$

The coherent length

$$l_{\text{coh}} = \frac{\lambda \beta \sqrt{\varepsilon_0}}{1 - \beta \sqrt{\varepsilon_0} \cos \vartheta} \quad (11)$$

is defined here as the reciprocal of the momentum q_{\parallel} longitudinally transferred to the interface when a photon is emitted in the ϑ direction. In the classical analysis this corresponds to the length of the trajectory of the radiating particle that plays a role in the formation of the transition radiation (see, e.g., Ref. [3], [7]). For nonrelativistic particles $l_{\text{coh}} \sim \lambda \beta \sqrt{\varepsilon_0}$ the condition (10) at $\cos \vartheta > \beta \sqrt{\varepsilon_0}$ is practically always weaker than the condition (9); for relativistic particles the condition (10) can be much stronger than the condition (9).

To illustrate this, consider the simplest expressions for spectral energy densities of transition radiation at normal electron incidence (particle velocity directed along the z axis) on statistically rough surfaces described by the function for the distribution of deviations of the surface points from the plane $z=0$.

In many cases the distribution of the deviations is approximated by a normal distribution (the Gauss law). For the height distribution density $f(x, y)$ one uses the two-dimensional normal distribution:

$$W(f(\vec{r}), f(\vec{r}')) = \frac{1}{2\pi(1-F)^2} \exp \left\{ -\frac{f^2(\vec{r}) - 2Ff(\vec{r})f(\vec{r}') + f^2(\vec{r}')}{2f_0^2(1-F^2)} \right\}, \quad (12)$$

where $W(f(\vec{r}), f(\vec{r}'))$ is the probability that at two points defined by radius vectors $\vec{r}(x, y)$ and $\vec{r}'(x', y')$ the heights of the surface turn out to be equal to f and f' .

In (12), the interface is defined by two parameters, the mean squared deviation of the heights $f_0^2 = \overline{f^2}$ (the bar denotes averaging over the surface) of the roughness from plane $z=0$, and the correlation coefficient F . The correlation coefficient is defined as the mean value of the product of the heights in two spatially separated points $\vec{r}(x, y)$ and $\vec{r}'(x', y')$

$$\overline{f(\vec{r})f(\vec{r}')} = f_0^2 F\left(\frac{x-x'}{l_x}, \frac{y-y'}{l_y}\right). \quad (13)$$

Since f_0^2 is the mean squared height, the correlation coefficient F at $\vec{r} = \vec{r}'$ is equal to unity. If the distance between the points \vec{r} and \vec{r}' exceeds the characteristic lengths l (called the correlation radii) for which the height correlation vanishes, the function F tends to zero. In most papers on light scattering by statistical inhomogeneities of a surface, the following expression is used for the correlation coefficient:

$$F = \exp\left(-\frac{|\vec{r} - \vec{r}'|^2}{l^2}\right) \quad (14)$$

for which the correlation radius l is the distance over which the correlation decreases by a factor e . The fact that the correlation function depends on the coordinate difference expresses the statistical homogeneity of the interface. On the other hand, in the case of statistically isotropic surfaces we have $l_x = l_y = l$. We shall consider only such interfaces.

We insert (6) into (2) and average the latter over an assembly of rough surface using the distribution function (12). We obtain simple expressions for the spectral energy densities of the transition radiation with parallel polarization (the electric vector lies in the radiation plane that contains the wave vector \vec{k} of the radiated quantum and the normal to the plane $z=0$) and perpendicular polarization (the electric vector is perpendicular to the plane of radiation).

In the case of weak roughness, when the following inequality is satisfied

$$f_0^2 \ll l_{\text{coh}}^2 \quad (15)$$

under the condition

$$\frac{l^2}{4\lambda^2} \sin^2 \vartheta \ll 1, \quad (16)$$

and when the inequality is also fulfilled

$$\frac{l^2}{4\rho^2} \ll 1, \quad (17)$$

i.e., when the transverse dimension of the field of the particle is large compared with the correlation radius, we obtain for the spectral densities of the transition radiation energy

$$I^{\parallel} = \left(1 - \frac{f_0^2}{l_{\text{coh}}^2}\right) I_{\text{PL}} + I^{\perp} \cos^2 \vartheta, \quad (18)$$

$$I^{\perp} = I_{\text{PL}} \frac{f_0^2}{l_{\text{coh}}^2} \cdot \frac{l^2}{4\rho^2} \frac{(1 - \beta^2 \varepsilon_0 \cos^2 \vartheta)^2 \ln(2\rho/l)}{\sin^2 \vartheta (1 - \beta^2 \varepsilon_0) (1 - \beta^2 \varepsilon_0 - \beta \sqrt{\varepsilon_0} \cos \vartheta)^2}.$$

In the case of strong roughness, when the inequality inverse to (15)

$$f_0^2 \gg l_{\text{coh}}^2 \quad (19)$$

is satisfied and under the conditions

$$\frac{L^2}{4\lambda^2} \sin^2 \vartheta \ll 1, \quad (16')$$

$$\frac{L^2}{4\rho^2} \ll 1, \quad (17')$$

where $L = l(l_{\text{coh}}/f_0)$ is a certain effective dimension smaller than the correlation radius l by a factor l_{coh}/f_0 , we obtain

$$I^{\parallel} = I^{\perp} \cos^2 \vartheta,$$

$$I^{\perp} = I_{\text{PL}} \frac{l^2}{f_0^2} \frac{(1 + \beta \sqrt{\varepsilon_0} \cos \vartheta)^2 \ln(2\rho/L)}{4 \sin^2 \vartheta (1 - \beta^2 \varepsilon_0 - \beta \sqrt{\varepsilon_0} \cos \vartheta)^2}. \quad (20)$$

Now we analyse the expressions (18). For nonrelativistic particles ($\beta \sqrt{\varepsilon_0} \ll 1$, $\rho \sim \lambda \beta \sqrt{\varepsilon_0}$, $l_{\text{coh}} \sim \lambda \beta \sqrt{\varepsilon_0}$), the transition radiation formulas change in order of magnitude for the emission angles

$$\sin \vartheta \lesssim \frac{f_0 l \sqrt{\ln(2\lambda \beta \varepsilon_0/l)}}{2\lambda^2 \beta^2 \varepsilon_0}.$$

If the foregoing inequality is satisfied as well as (15) and (17'), which limit β from below, then not only the depolarization of the radiation observed is complete but also the intensity of the transition radiation greatly exceeds the radiation on a flat boundary. If the inverse inequality is satisfied the radiation intensity is

$$I = I^{\parallel} + I^{\perp} = I_{\text{PL}} \left(1 - \frac{f_0^2}{l_{\text{coh}}^2}\right).$$

In the case of strong roughnesses for nonrelativistic particles at emission angles

$$\sin \vartheta \lesssim \frac{l}{2f_0} \sqrt{\ln\left(\frac{2f_0}{l}\right)}$$

the transition radiation is likewise completely depolarized and exceeds the radiation from a flat boundary. When the opposite inequality is satisfied, the intensity of the radiation is suppressed compared with the intensity on the plane interface. The same condition with a coefficient of the order of unity holds also for

relativistic particles in the last case. As for the emission of relativistic particles from interfaces with weak inhomogeneities, the emission angles at which the enhancement effect is observed are decreased in comparison with the above expression for rough surfaces by a factor $(1 - \beta \sqrt{\epsilon_0} \cos \vartheta)^2$.

Thus, the investigation of the angular dependence of the intensity of transition radiation of nonrelativistic particles, for both strong and weak roughnesses, can yield valuable information on the values of f_0 and l that characterize the surface.

If the inequality inverse to (17) is satisfied (the transverse dimensions of the field are small compared with the correlation radius)

$$\frac{l^2}{4\rho^2} \gg 1 \quad (21)$$

we have

$$\begin{aligned} I^{\parallel} &= I_{\text{PL}} + I^{\perp} \cos^2 \vartheta, \\ I^{\perp} &= I_{\text{PL}} \frac{f_0^2}{l} \frac{2(1 - \beta \sqrt{\epsilon_0} \cos \vartheta)^2}{\sin^2 \vartheta (1 - \beta^2 \epsilon_0 - \beta \sqrt{\epsilon_0} \cos \vartheta)^2}. \end{aligned} \quad (22)$$

These expressions are valid for weak as well as for strong roughness, but in the case of strong roughness the validity conditions are changed: (16) is replaced by (16'), and (21) takes the form

$$\frac{L^2}{4\rho^2} \gg 1. \quad (21')$$

For the emission angles

$$\sin \vartheta \lesssim \sqrt{2} \frac{f_0}{l} (1 - \beta \sqrt{\epsilon_0} \cos \vartheta),$$

the radiation is completely depolarized and exceeds the transition radiation from a plane interface; at large angles, the radiation tends to that from a flat interface for both strong and weak roughness.

Thus, the equations presented give a clear idea of the influence of the roughness on the transition radiation.

From the analysis of the expressions derived for the spectral energy densities of the transition radiation it follows that two cases can be singled out. In the first, the transverse dimensions of the particle field are large compared with the correlation radius. In expressions for weak roughness for the parallel component of the spectral energy density of the transition radiation, the first term leads to an equation for I_{PL} with the additional factor $(1 - f_0^2/l_{\text{coh}}^2)$ that influences the radiation because of effects connected with the longitudinal dimensions of the inhomogeneities. The second term leads to an additional contribution to the radiation, compared with a plane interface, on account of transverse effects. In the expressions for strong roughnesses there should be no limiting transition in the

expression for a plane interface. The difference from the case of weak roughness lies in the fact that the spectral density of the transition radiation energy in the case of perpendicular polarization exceeds the spectral energy density in the case of parallel polarization by a factor $\cos^{-2} \vartheta$. In the second case the transverse dimensions of the particles field are small compared with the correlation radius. In this case the equations for weak and strong roughnesses do not differ from each other and go over in the limit into the equations for a plane interface. The expression for I^\perp are larger the larger is f_0^2/l_{coh}^2 . The additional contribution I^\parallel to the radiation is due mainly to transverse effects.

Summing up the results of all our calculations both for different models of regular surfaces and for surfaces with nonregular roughnesses, we will present the main results below.

The interface roughness results in a depolarization of transition radiation which increases with the corresponding increase in roughness height and the decrease in the ratio of the transversal sizes of the roughnesses and their heights. The depolarization also increases with the increase in particle incidence angle and the decrease in radiation angle. For the case of normal particle incidence there is a radiation which is depolarized completely at the observation angle $\vartheta = 0^\circ$.

Depending on the parameters of the interface roughnesses and the characteristics of the transition radiation, the curves for the parallel component of the transition radiation on rough interface fall both above and below the parallel component of the transition radiation from a plane interface in the spectral distribution of the radiation. In the case of oblique incidence of the particle onto the target the parallel component for rough interface is located below the parallel component for plane interface and the discrepancy amounts approximately to 20% at large radiation angles. At small radiation angles the intensity of the parallel component of radiation exceeds the intensity of the transition radiation in the case of plane interface. In some cases the intensity of the non-polarized part is greater than the total intensity of transition radiation from a plane interface at small radiation angles.

With increasing wavelength, the spectral energy density of the parallel component decreases. For the non-polarized part of the radiation a smooth drop with a corresponding increase in wavelength is observed in the case of statistically rough interface.

At normal incidence there is a symmetric pattern of angular distribution of radiation relative to the direction of electron incidence. This symmetry is upset with the change in incidence angle.

And now, to end the paper it will be worthwhile to sum up the current state of the theory and experiment to visualize the possible trends of further development. For not very big angles of electron incidence onto the target when the transition radiation mechanism is predominant, the comparison of theory with experiment shows satisfactory agreement. For grazing angles, when experiment [2] shows that the intensity of radiation reaches values by one order of magnitude greater

than maximal intensity of transition radiation on a plane interface, a radiation analogous to the Smith-Purcell case prevails. The grazing angles case calls for separate consideration. However, the theory of the problem is still far from completion since the surfaces considered approximate the real interface only to a small extent. Only for some models it is possible to bring the theory to the estimation formulas. Experimental investigation of the problems treated above is also in an unsatisfactory state. To establish an unambiguous correspondence between theory and experiment the non-polarized part of the radiation has to be considered in detail on interfaces with a known structure of roughness. Besides, experimental investigations at small angles of radiation may prove highly interesting since the theory predicts great values of the radiation intensity at such angles.

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